Prisoner's Dilemma with Semi-synchronous Updates: Evidence for a First Order Phase Transition

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Abstract. Emergence of cooperation in self-centered individuals has been a major puzzle in the study of evolutionary ethics. Reciprocal altruism is one of explanations put forward and prisoner's dilemma has been a paradigm in this context. Emergence of cooperation was demonstrated for prisoner's dilemma on a lattice with synchronous update [*Nature,* **359***, 826 (1992)*]. However, the cooperation disappeared for asynchronous update and the general validity of the conclusions was questioned [*PNAS,* **90***, 7716 (1993)*]. Neither synchronous nor asynchronous updates are realistic for natural systems. In this paper, we make a detailed study of more realistic system of semi-synchronous updates where pN agents are updated at every time instant. We observe a transition from all-defector state to a mixed state as a function of p. Despite being transition from absorbing state, our studies indicate that it is a first order transition. Furthermore, we used damage spreading technique to demonstrate that, the transition in this system could be classified as a frozen-chaotic transition.

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1. Introduction

In ecological and social systems, game theoretical models have been immensely successful and have emerged as a standard models in certain cases. One of the interesting applications of game theory in these systems has been explanation of emergence of co-operative behavior. It was suggested that cooperation could be sustained if there is repeated interaction and spatial structure. Prisoner's dilemma on a lattice has emerged as a paradigm in this context [1]. We will be analyzing a stochastic variant of the same in this work.

"Prisoner's Dilemma" (PD) is an interesting problem in game theory where cooperation between agents is strictly dominated by defecting and the possible equilibrium is that everyone defects. This is despite the fact that cooperation will lead to better average payoff for everyone. However, cooperation can arise in an iterated version of the game. This happens mainly due to threat of punishment for defection in the next round. If game theory in traditional sense is applied to social system and if we assume common knowledge and perfect rationality for each agent, cooperation is not a viable option. Refining these assumptions makes cooperation possible. In iterated games on a lattice, agents have knowledge only about their nearest neighbors and the interaction is repeated. Cooperation is indeed observed in these models. Thus it is claimed that the emergence of cooperation, which seems counterintuitive in the context of Darwinian evolution but observed in several biological and social contexts, is explained by this model. In evolutionary games, the strategies are built in trial-and-error fashion and more successful strategies are preferred in course of time. This procedure leads to dynamics automatically. Obviously, societies comprise of several agents who interact through a complex network of acquaintances. This fact has motivated the analysis of iterated version of PD on variety of networks. These studies have interesting consequences in the social, biological and economic systems [2, 3, 4]. The choice of underlying network depends on the problem being addressed. In this work, we study

the PD on a two dimensional lattice. The reason is that evolutionary games on 2-D networks is a relevant and popular choice for several biological networks [5]. (We must mention that recently other spatial structures have also received a fair share of attention [6, 7, 8, 9, 10, 11, 12].) An extensive survey of modeling using game theoretical models in ecology can be found in a recent book [13]. In this paper, we will focus on studying various phases in this system and nature of dynamic phase transitions between these phases.

In this model, the agents sitting on the nodes of two-dimensional lattice update their strategies in a synchronous manner and payoffs are computed using the payoff matrix of PD. The major finding is that on introduction of spatial structure, there is emergence of cooperative behavior among selfish individuals [5]. However, this model was strongly criticized by Huberman and Glance and also by Mukherji *et al*[14, 15]. They studied the robustness of these results with respect to stochastic fluctuations and concluded that several of the conclusions do not hold in presence of stochastic fluctuations. One stochastic variant studied in both these works has been relaxation of requirement of synchronous update. In modeling physical phenomena, it is difficult to say whether synchronous updating scheme is more natural than asynchronous scheme or vice versa. Models of traffic flow have more realistic properties if updated in synchronous mode [16]. However, if we are modeling physical phenomena in realm of equilibrium statistical physics, asynchronous updates offer better results [17]. There could be phenomena where an intermediate mode could be more realistic. In above model, certain degree of asynchronicity in updating strategies is relevant and possible and this variant is studied in detail in this work. We note that the differences between synchronous and asynchronous updates have been a topic of recent interest in statistical physics community. Apart from game theory [18], it is studied in the contexts of boolean networks [19, 20], coupled map [21], neural Networks [22], Monte Carlo processes [23] and biological networks [24, 25]. It has even been studied in the context of equilibrium models [17] such as Ising Model. In above work, nonequilibrium phase transition, induced by introducing semi-synchronous updating in Ising model, is studied. In the context of game theory, there is a clear physical motivation for such study since it is unlikely that even a strategy with slightly more payoff is deterministically copied by everyone. Certain degree of stochasticity in updating of strategies is indeed possible in these systems. Thus there is need for a detailed study of systems which do not evolve in a fully synchronous or asynchronous manner. Hence, we make a detailed study of the evolution for semi-synchronous update where pN members update strategy every instant in PD on a lattice.

For completeness, we define the 2-person PD game in its classic form: this game describes the confrontation between two players, each of whom may choose either to cooperate (strategy C), or defect (strategy D), at any confrontation. If both players choose C, they get a pay-off of magnitude R each; if one player chooses to D while the other chooses C, the defector player gets the biggest pay-off T, while the other gets S; if both players defect, they get pay-off P. In this game, the pay-off values must satisfy

the inequalities $T > R > P > S$ and $2R > S + T$. For such choice of parameters, the paradox is evident. Each player is tempted to defect, but they would be worse off if both defected and total payoff for both together would be higher if they co-operated in stead. However, best payoff for an individual player is obtained when he defects while the other player cooperated.

In ecological context, Nowak and May simulated this system with choice of parameters $R = 1$, $T = b$ $(1.0 < b < 2.0)$ and $S = P = 0$ [5, 26]. They believe that most of the interesting behavior is reproduced for this choice of parameters. They studied the PD on a two dimensional array with synchronous updating. They explored the asymptotic behavior of the game for various values of the b. Here players interact with their local neighbors through simple deterministic rules and have no memory of past. They found that, the dynamical behavior of the system depends on the parameter b. For a range of values of b $(1.0 < b < 2.0)$, system reaches a steady state with a non-zero fraction of cooperators. They concluded that spatial structure and repeated interactions promote cooperation in the PD.

This synchronous updating came in for heavy criticism. It was argued that, the global clock in the social systems are very rare and the probability of two events evolve exactly at same time has measure zero. It was also argued that more realistic modeling would involve updating the system by individual. Thus only one player should be updated at each time step. This type of updating is called asynchronous update. As we mention previously, Huberman and Glance studied PD game with same parameters of Nowak and May under the asynchronous updating rule [14]. A similar argument was made by Mukherji *et al.* [15] and Masaki and Mitsuo [27]. In this case, the system rapidly converges to steady state where all the players become defectors. They argued that the previous results about emergence of cooperation are not generic. Nowak et al [28, 29, 30], replied that if they study the behavior in entire parameter space, cooperation is observed even for asynchronous update for some choice of parameters. They found that, the two updating rules are similar for a some values of b, but for $(1.8 < b < 2.0)$ the two updating rules lead to different steady states. They also argued that discrete time is appropriate for many biological situations where interaction phase is followed by reproduction phase. Thus synchronous update is more relevant and realistic biologically. However, neither completely synchronous nor completely asynchronous updating is realistic in natural processes and there are bound to be stochastic fluctuations in updating. There have been attempts to interpolate between these two cases [17, 18, 19, 20, 21, 22, 23, 24, 25]. In this work, we make a detailed study of PD on a 2-D lattice with semi-synchronous updates from the viewpoint of statistical physics and investigate how generic the results are.

We will make a detailed study of the observed phases for semi-synchronous update and present a phase diagram. The phases of interest in this system are an all-defector state and a mixed phase with cooperators and defectors. In particular, we will study the transition between these two phases as a dynamic phase transition.

In statistical physics, a lot of effort is devoted for finding the order of transitions

and critical exponents in case of continuous phase transitions. The reason is that the critical behavior lets us distinguish between essential and not so essential details of the system. Idea of universality in the theory of phase transitions has allowed us to see how seemingly disparate models have common underlying features. Thus it is important to study the transitions in detail. All agents becoming C or D, is an absorbing state while coexistence of D and C can be considered as an active phase. Hence, this system characterized by two absorbing state, all defectors state and all cooperators state. It has been long argued for that all one component systems undergoing from active phase transition to a unique absorbing state have a phase transition in the class of directed percolation (DP) if the order parameter is a scalar and there are no extra symmetries or conservation laws and the interaction through short-range [16]. Also, Most system with multiple absorbing state found to fall in the DP class [31]. We will study the veracity of this conjecture. Firstly we need to establish the order of the transition. The first (second) order phase transitions in equilibrium systems are characterized by discontinuities in the first (second) derivatives of free energy e.g., the internal energy and order parameter. This singularities at a first order phase transition are due to phase coexistence and there are no critical exponents.

For PD game system, it was found that, for different choice of updating rule transition to all-defector state on variation of parameter b is in the class of directed percolation [32, 33]. We will study the Nowak and May's system under the variation of probability p of update. *We observe that the transition is not continuous.* We must also mention that there are several known exceptions in nonequilibrium systems which exhibit a discontinuous transition to absorbing state [34, 35, 36, 37, 38, 39, 40].

Our updating strategy is as follows. We allow every player to update his strategy with probability p in each Monte Carlo step. Under variation of this probability from $p \to 0$ (asynchronous) to $p = 1$ (synchronous) in thermodynamic limit, we study the effect of the updating scheme on the behavior of the system. In second section of the paper, we establish that the transition is indeed a first order transition. In the third section, we carry out a damage spreading analysis to study the dynamical phase digram and critical behavior.

2. The Model and Simulation

We investigate PD on the two dimensional lattice of size L with evolutionary dynamics. The agents on each site of lattice can choose only two strategies $D = 0, C = 1$. D corresponds to defector while C corresponds to cooperator. (The defectors and cooperators have also been viewed as dead and living sites in some applications.) We employ fixed boundary conditions and assume that the agents have no memory of the past. Initial configuration consists of 30% defectors and 70% cooperators distributed randomly on the lattice. (We checked other initial conditions as well. We varied the density of defectors between 10% to 50% and found that the asymptotic stationary state did not change.) Every agent interacts with eight nearest neighbors and self. We set the pa-

rameters $T = b(b = 1.83)$, $R = 1$ and $S = P = 0$. The pay-off matrix is

	C	D
C	1	0
D	b	0

The players interact simultaneously and independently of each other. Their pay-off is the sum of the pay-offs from all nine interactions (with neighbors and self). Generally, each player updates its strategy by imitating the strategy of most successful agent in the neighborhood. The main variation in this work is as follows: At every time-step, each player updates his strategy by adopting the strategy of the most successful neighbor *with probability* p. For asynchronous update, only one agent updates strategy at every instant. This could be compared with evolution with $p = 1/N$ where N is total number of agents. On the other hand, all agents update their strategy for synchronous update and the above rule for $p = 1$ is same as synchronous update which is widely studied. We vary the value of p from $p \to 0$ to $p = 1$ interpolating between asynchronous and synchronous update. The phases of interest are all-defector and mixed phases. We study the domains in $p - b$ plane, where almost all initial conditions lead to one of these phases.

We carried out a detailed investigation of this system for parameter values in the range $0.0 < p < 1.0$ and $1.0 < b < 2.0$. The corresponding phase diagram is shown in Fig. 1. For values of b in the range $1.8 < b < 2.0$, the final phase depends on the value of p. In this work, we study the nature of the phase transition observed on varying p. The density of cooperators ρ_c is an obvious order parameter for describing the transition since it is zero in an all-defector state and positive for mixed state.

We compute the value of ρ_c in steady state for different values of updating parameter p. We also investigate the same for different lattice sizes L. We observe that, when we change p from $p \to 0$ to $p = 1$, the system switches from an all-defector absorbing state with $\rho_c = 0$ to active phase with $\rho_c > 0$. We find a clear evidence of the metastable state near the transition region. In Fig. 2, we plot the lifetime (average time taken by the system to reach an all-defector absorbing state) as a function of updating probability p for several values of L. For smaller values of lifetime ($< 10^6$), we average over 100 configurations and for larger lifetimes, we average over 10 configuration. Depending on the value of the parameter p we can distinguish between three regions:

- For a long range of the values of the parameter $p < p^*$, the system rapidly converge to absorbing all-defector state. For $p > p^*$ the time required to reach this absorbing state grows abruptly compared to p values smaller than p^* Fig. 2. In this region, the lifetime approximately equally for all values of parameter p and equal to the lifetime in asynchronous update. Thus one could say that the behavior of system is analogous to one obtained in purely asynchronous updates. As shown in Fig. 2, the value $p*$ converge to finite value at thermodynamic limit.

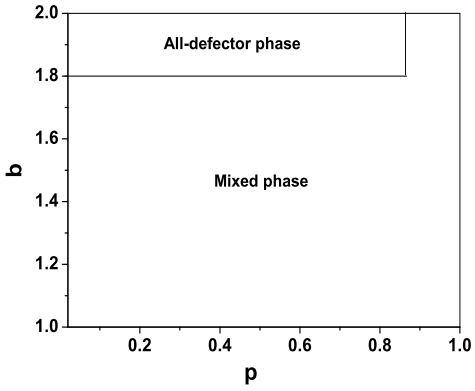

Figure 1. Schematic phase diagram of the semi-synchronous PD game. We plot the phases observes as a function of p (probability of update for any given site) and b (temptation to defect). For the values of $1.8 < b < 2.0$ system switches from absorbing state (all defectors) to an active state where cooperators and defectors coexist. Simulations are carried out for $L = 60$, and initial 1000 time-steps are discarded. We average over 100 different initial conditions.

- There is very sharp range of the value of parameter $p^* < p < p^{**}$ where the system falls into metastable state. The mixed state in which both cooperators and defectors are present is extremely long-lived and system falls in an all-defector state after a very long time. The average time taken by the system to reach this absorbing state increases with updating probability p till it reaches a very large value as $p \to p^{**}$. For any value $p > p^{**}$, the system remains in the active phase. As $p \to p^{**}$ the system spend longer time in the active phase before it collapse eventually to its absorbing state. We found that, for the best fit the lifetime as a function of $(p^{**} - p)$ is an exponential decreasing fit. (See Fig. 3)
- For values of $p > p^{**}$, we indeed found that the system saturates with finite number of cooperators. We would like to assert that, as the system crossover the metastable state region toward this region the order parameter ρ_c exhibits a certain jump in its value (See Fig. 4).

Appearance of metastable state in this system shown us some similarity to equilibrium discontinuous transition. It is known that, the discontinuous transition accompanied usually by metastability.

To confirm that for any value of $p < p^{**}$ the system collapses to an absorbing state, we plot the time evolution of density of cooperators $\rho_c(t)$ as a function of time t in Fig. 4. We show the behavior of $\rho_c(t)$ as a function of t for various values of p for a system of size $L = 60$. We average over 10^3 different initial conditions. The finite number of cooperators are observed for some time (depending on the value of p. The time taken to reach absorbing state increases exponentially as $p \to p^{**}$ as mentioned previously) followed by a collapse to an absorbing state. As $p \to p^{**}$, the curves in Fig. 4 becomes flatter (the system need longer time to approach absorbing state) till the system reaches

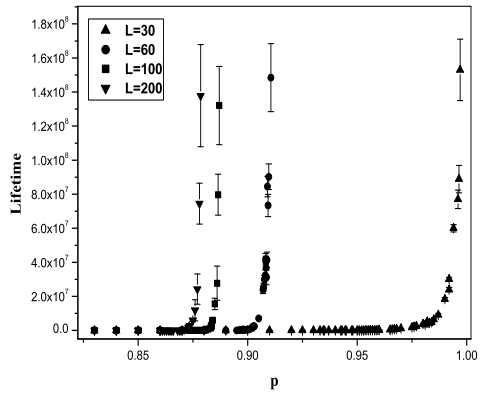

Figure 2. The average time required for the system to reach its absorbing state (lifetime of coexistence state) is plotted as function of updating probability p for different lattice sizes. The lifetime changes abruptly and diverges near the critical point.

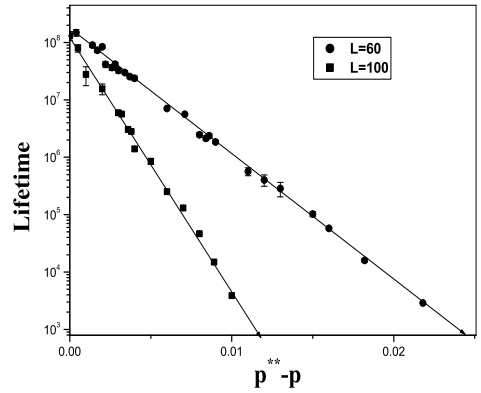

Figure 3. The semi-log plot for the lifetime of coexistence stat as a function of $(p^{**}-p)$ is plotted for $L = 60$ and 100 in the metastable state. (We note that p^{**} is the value of updating probability p above which system reaches a saturated state with coexistence of cooperators and defectors.) The solid line shown the exponential fit for the data.

its steady state at $p > p^{**}$. Fig. 4 shows that, when the steady state of the system changes from an all-defector state to coexistence state, there is certainly a jump in the value of the order parameter ρ_c. Metastability and long time required by the system to reach its steady state make it very difficult to locate the critical value of p above which the system reaches its stable active phase. We run the programs for very long times so that correct estimates can be made. In the Fig. 5, we present the order parameter ρ_c as function of the parameter p. We used lattice of size $L = 100$, we averaged over 10^2 samples after discarding 10^6 transient time. The figure shows the clear jump in the value of order parameter ρ_c.

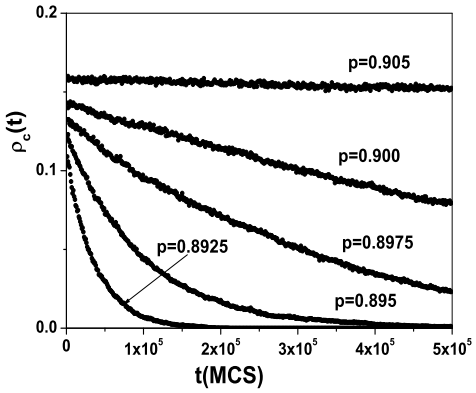

Figure 4. The time evolution of the density of cooperators $\rho_c(t)$ for $L = 60$ for different values of the parameter p near the steady state region. On increasing value of p from $p = 0.8925$ (bottom curve) to $p = 0.905$ (top curve) system needs more time to reach absorbing state. It is clear that at $p = 0.905$, the curve becomes flat, and system reaches the active coexistence phase asymptotically. We start simulation with 70% cooperators in all cases. There is a clear jump in the asymptotic value of $\rho_c(t)$.

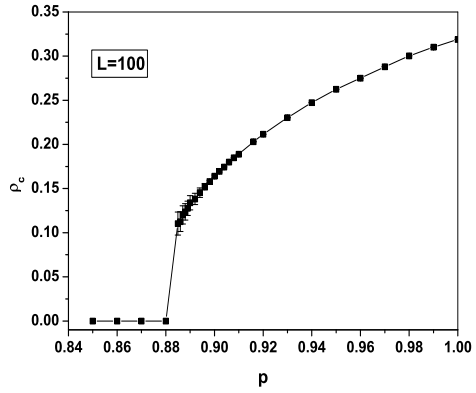

Figure 5. The asymptotic value of order parameter ρ_c as function of p for lattice size $L = 100$. A clear jump in the value of order parameter at the critical point indicates a first order phase transition.

This result of a clear jump in the order parameter value at the transition point coupled with a presence of metastable state indicate that the model undergoes a first order phase transition between the active and absorbing phase.

2.1. Analysis using Binder cumulant

As mentioned above, presence of long-lived metastable states makes it very difficult to locate the critical point and the kind of phase transition since one could always doubt if the jump in order parameter is true. Fortunately, fourth-order reduced Binder cumulant offers a precise tool which is very sensitive to the nature of the phase transitions. The

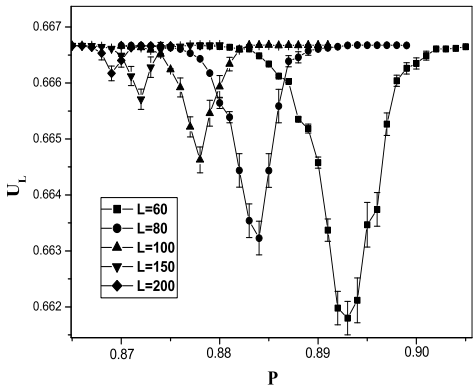

Figure 6. Binder cumulant U_L is plotted as function of the probability p for different lattice sizes. The presence of a minimum suggests that the transition is of first order.

fourth-order reduced Binder cumulant of the order parameter ρ_c is defined [41, 42] as:

$$U_L = 1 - \left\langle \rho_c^4 \right\rangle / 3 \left\langle \rho_c^2 \right\rangle^2, \tag{1}$$

Systematic analysis of Binder cumulant U_L has been successfully used to determine the order of the phase transition in several equilibrium phase transition [43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53]. In the second order phase transition, $U_\infty = 2/3$, at the transition point in the thermodynamic limit. On the other hand, for first order transitions, it has a minimum at transition point. The reason is as follows. For a continuous phase transition, the distribution of values of order parameter is always a Gaussian, the position of which keeps changing. For a first order transitions, distribution is different. Here, we have phase coexistence and distribution is superposition of two Gaussian centered at values corresponding to each phase [41, 42, 43]. The quantity U_L is not well defined numerically when $\rho_c \to 0$ which is the case here. To overcome this difficulty we follow [43, 44] and add an arbitrary fixed constant to all values of the order parameter ρ_c, (we fixed it to be equal 0.001 in this work), that is rigidly shift the probability distribution of the order parameter away from zero.

We have plotted Binder cumulant U_L as function of probability p for various $L's$ in Fig. 6. We average over 150 configurations and 5×10^6 iterations on discarding 10^4 transients. A clear minimum in the value U_L shows that the transition is of first order. The value of probability p that corresponds to a minimum of the Binder cumulant U_L in the thermodynamic limit is the critical point.

It is well known that for the first order phase transition, finite-size scaling theory predicts rounding and shifts of the critical point to be inversely proportional to the volume, L^d in d dimensions [41, 42, 54]. We have plotted $p_c(L)$ as function of L^{-2} in Fig. 7. We found an excellent linear behavior which matches with finite- size scaling prediction. In the thermodynamic limit the value of $p_c = 0.8678(6)$ for the best fit.

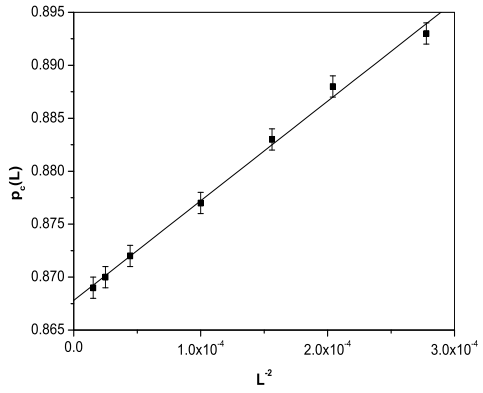

Figure 7. The critical point $p_c(L)$ is plotted as a function of L^{-2}. The data came from the same simulations results as in Fig. 6. An excellent linear behavior matching with finite-size scaling predictions for first order transition is obtained.

2.2. Hysteresis

Hysteresis effect is a characteristic feature of first order phase transitions. The magnetic systems show a first order transition on varying magnetic field but a second order transition on varying temperature. Thus there is a hysteresis on variation of magnetic field, but not on varying temperature. Thus is an useful tool to distinguishing first order phase transition from continuous phase transition. The reason for hysteresis in first order phase transition is due to the coexistence of two phases. However, there is one difficulty in studying hysteresis in transitions leading to an absorbing state. If the system falls in an absorbing state, it can not come out. Thus we need to suitably modify the model. Hence, we use the spontaneous creation method (SCM) [35, 55, 56, 57]. The SCM overcomes this difficulty by allowing for small nonzero ϵ concentration of active site to survive. For the second order phase transition, this spontaneous creation rate of active sites will destroy the phase transition. However, for first order phase transition a small spontaneous creation does not change the nature of the transition. It only turns the absorbing state into a fluctuating state of average density ϵ.

We carry out simulations for $L = 200$ and allow $\epsilon = 0.0005$ fraction of active sites to survive. We vary p stepwise in steps of $\Delta p = 0.005$. We record the value of the density of cooperator ρ_c after t_r update. We average over 100 loops and plot the density of cooperators ρ_c as a function of p in Fig. 8, for different relaxation time t_r. The evidence of hysteresis effect confirms our conclusion that the transition in the above system is a first order transition.

3. Damage Spreading Analysis

The damage spreading technique was suggested for first time by Kauffman in the context of biological systems [58]. Recently, This concept attracted much attention and applied

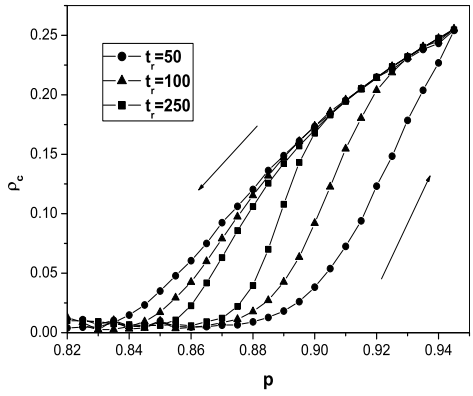

Figure 8. Hysteresis loops of the density of cooperators are obtained by SCM method for lattice size $L = 200$. They are obtained for different relaxation times t_r and it is clear that the loop area decreases for higher t_r. The loops are generated counterclockwise.

to analyze several dynamical systems such as cellular automata [59], kinetic Ising models [60] and surface growth model [61].

In the damage spreading technique, we follow the time evolution of two almost identical configurations. The initial condition for the second configuration is same as the first configuration except for small perturbation. Now one studies the evolution of these configurations in time under the same dynamics and see how the initial perturbation (damage) propagates. In our case, we start the simulation system from a random initial configuration. We allow the configuration to evolve until reaches its steady-state. Let us label this configuration as σ^A (first copy). The σ^B (second copy) is created from saturated state of the first copy σ_A by carrying out a small perturbation or damage to the this copy. Now both the copies are evolved under same dynamics in the following sense. We update the strategy of any agent in system A with probability p. Whenever we choose to update (not to update) the strategy of agent in the first copy, we also update (do not update) the strategy of the agent in second copy. This synchronized updating ensures that same set of random numbers is used during updating. We study the time evolution of both configurations under this dynamics. Evolution of the configuration can be described by trajectories in the phase space. The question is how the difference between these initial conditions grows or decays as a function of time. In other words, the question is if the damage heals or spreads. If the two initially close trajectories quickly become different, it is generically called chaotic. In order to measure the difference between that two systems, a useful metric is given by the Hamming distance or damage define by [62]

$$D(t) = 1/N \sum_{i=1}^{N} \left| \sigma_i^A(t) - \sigma_i^B(t) \right| \tag{2}$$

where N is the number of sites of the system. The quantity $D(t)$ measures the fraction

of sites of configuration σ^A which have different strategy from those from configuration σ^B. In the thermodynamic limit, $D(t)$ may go to zero if damage heals completely while it may tend to a positive value if it does not heal. For chaotic system, we expect the damage to grow in time and reach certain asymptotic value. However, in the so-called frozen phase $D(t)$ will go to zero [63, 64, 65].

Nowak and May, in their original paper, do not exactly make a systematic study of damage spreading as done in present work. However, they empirically simulate different configurations and call a phase chaotic if a slightly perturbed initial condition leads to a very different state. They state that the steady state of PD with synchronous update is chaotic for the parameter range $1.8 < b < 2.0$ [5]. This is very similar to the definition of chaotic phase in damage spreading studies. In this work, we will make a systematic study of damage spreading in PD as a function of updating probability p. We would like to mention that similar studies were carried out for the Stochastic Game of Life (SGL) by Monetti and Albano. They found a first order transition as a function of stochasticity and a rich dynamic critical behavior in the system[66]. Since our system is similar, we expect an analogous dynamic behavior in our system.

3.1. Analysis of Damage Spreading

We followed the same updating role and initial values as described in the previous section. We start our simulation with 70% cooperators and 30% defectors distributed randomly on the sites of square lattice. As mentioned before, we update the strategies with probability p and wait till the system reaches its steady state and we label this configuration as σ^A. The second configuration σ^B created with small damage in the central sites of configuration σ^A. The evolution of damage spreading is computed as a function of time for different update probabilities. We have plotted $D(t)$ as a function of t in the active phase for various values of p in Fig. 9(a) In the log-log scale the damage $D(t)$ evolves linearly as function of t until it saturates to a finite value of damage after some time. In the Fig 9(b), we have plotted the average asymptotic value of damage $\langle D(\infty)\rangle$ for different values of the parameter p for lattice of size $L = 200$. It is clear that below a certain threshold probability, the damage reaches zero asymptotically. This threshold probability is very close to p_c of the transition mentioned above. So, we conclude that the dynamical phase transition behavior in this system between the absorbing all-defector state and mixed state is intimately connected to frozen-chaotic transition in damage spreading.

We also study the effect of finite lattice size on damage spreading. In Fig. 10 (a), we plot the damage $D(t)$ as function of t for various value of lattice size L for $p = 1$. The behavior obtained is very similar to one observed in SGL [66]. It is the evident that, the slopes and saturation values of all curves is independent of lattice size L. However, the time required for the system to reach the plateau $\tau(L)$ increases with L. The behavior

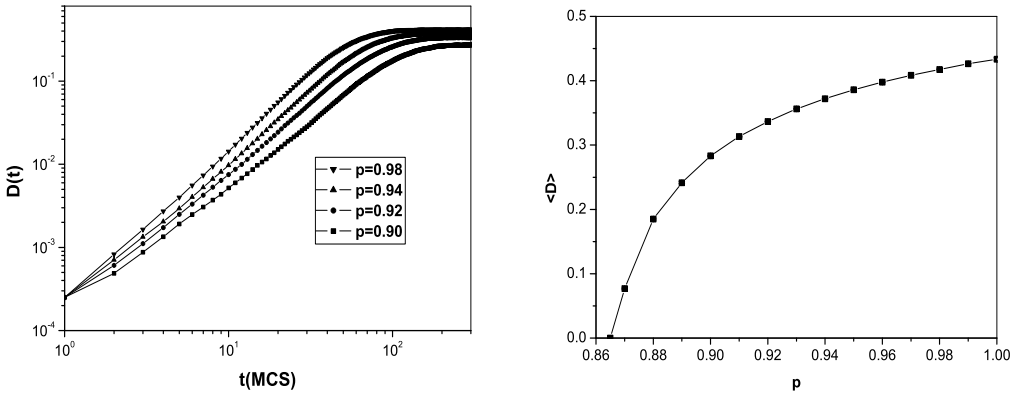

Figure 9. (a) Damage spreading $D(t)$ is plotted as a function of time for lattice size $L = 200$ for different values of parameter p. We average over 100 configurations. (b) The average asymptotic value of damage $\langle D \rangle$ for different values of the parameter p for lattice size $L = 200$

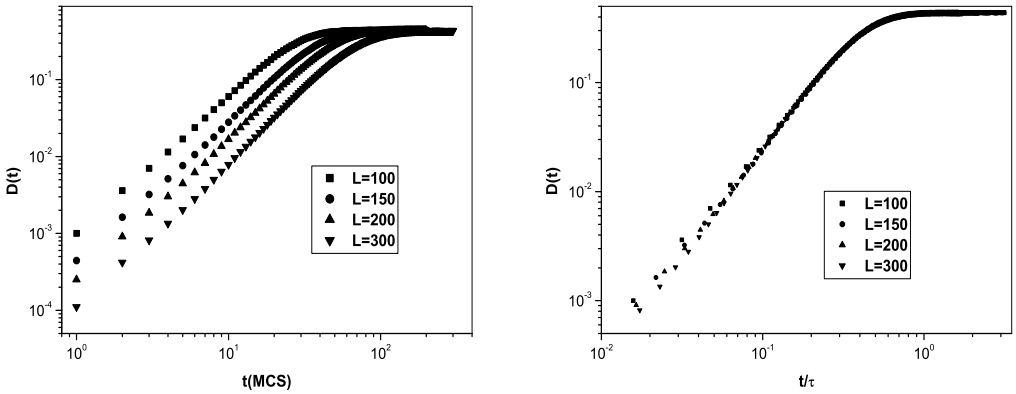

Figure 10. (a) Damage spreading $D(t)$ as function of t obtained for different lattice size L at fixed value of the parameter $p = 1$. (b) The damage $D(t)$ versus t/τ for the data shown in (a). The data for four different lattice size collapse onto a single curve.

can be summed by the scaling form [66]

$$D(x) \propto \begin{cases} x^{\alpha} & x \leq 1; \\ const & x > 1. \end{cases} \tag{3}$$

where $x = t/\tau(L)$ and α is an exponent. In Fig. 10(b), we show the scaled data. It is clear that the data for four different lattice sizes collapses to a single curve using above scaling form. We calculate the value of α for the different value of the parameter p in the Table 1 .

The average mean-square distance $R^2(t)$ over which the initial damage spreads from the center of the lattice toward the boundary is also calculated in following manner. We start the simulation at $t = 0$ with initial damage of one site at the center of the lattice.

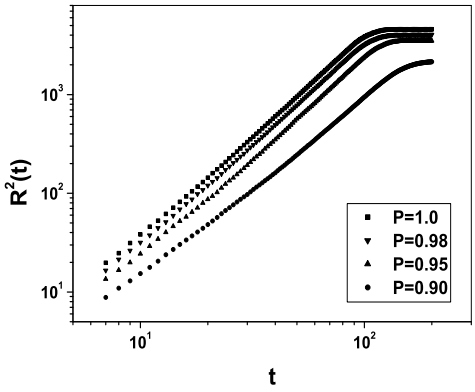

Figure 11. The average mean-square distance for epidemic spreading $R^2(t)$ is plotted as a function of t for lattice size $L = 199$ for different values of p.

Table 1. Values of the exponents α,β and d_f at the different value of the parameter p.

p	α	β	d_f
1.0	1.96(9)	2.03(7)	1.93(1)
0.98	1.88(7)	2.01(5)	1.87(1)
0.96	1.85(8)	1.98(9)	1.86(8)
0.92	1.64(2)	1.90(1)	1.72(6)
0.90	1.52(8)	1.82(2)	1.67(1)
0.88	0.97(8)	1.59(4)	1.22(1)

Lattice size is fixed at $L = 199$ and mean-square distance $R^2(t)$ is computed as a function of time t. We carry out the simulations for various values of parameter p. We average over 100 configurations and the results are plotted in Fig. 11. It is found that $R^2(t)$ and $D(t)$ exhibit a similar behavior. Thus the following scaling ansatz should hold [66]

$$R^2(x) \propto \begin{cases} x^\beta & x \leq 1; \\ const & x > 1. \end{cases} \tag{4}$$

where β is an exponent. The value of exponent β is tabulated as a function of p on Table 1.

The number of damaged sites is related to the spatial extent of the damage through the fractal dimension d_f of the damaged cloud, so

$$D(t) \propto R^{d_f}(t) \tag{5}$$

Then from the Eqs. (3) and (4) we get that

$$2\alpha = \beta d_f \tag{6}$$

We tabulate values of d_f in Table 1 using Eq. 6. While the fractal dimension d_f is found very close to 2 for synchronous update($p = 1$), it deviates appreciably for smaller

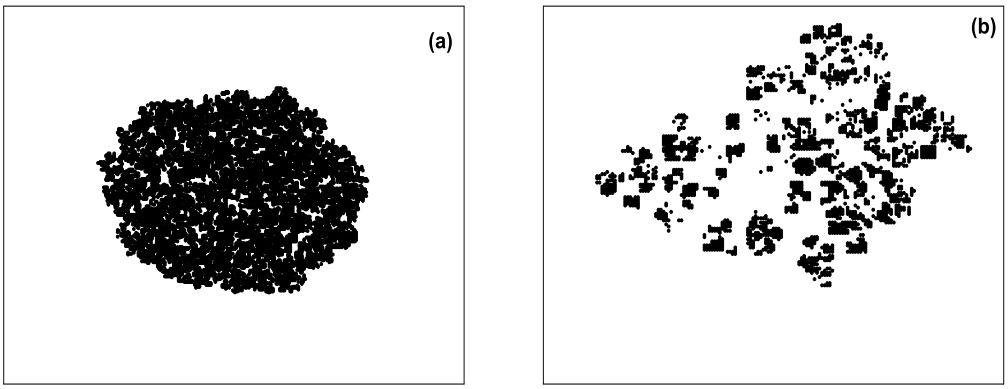

Figure 12. (a) Snapshot of damaged lattice sites at $t = 100$ for $p = 1.0$.(b) Snapshot of damaged lattice sites at $t = 100$ for $p = 0.88$. The initial damage is of one site at the center of 400×400 lattice.

values of p. For the value of p near the critical point p_c, the fractal dimension d_f is smaller. We find that the damage spreading near the critical point is certainly a fractal object. On the other hand, the snapshot in Fig. 12 shows that for synchronous update ($p = 1$), the damage spreading is approximately compact. Thus the damage spreading is fractal near the critical point and compact for synchronous update.

4. Conclusions

We have studied the PD on a $2 - D$ lattice with an update rule which interpolates between asynchronous and synchronous update as a function of the parameter p. Here each agent updates the strategy with probability p at each timestep. We observe that, this system crosses from mixed (active) phase to all defector (absorbing) phase when we vary the parameter p. We studied the time evolution of this system and found that this system exhibits long lived metastable state near the critical point. The order parameter of this system shows a clear jump at the critical point. We carry out detailed quantitative analysis to show that the the above transition s a first order transition. We confirm this result by studying the average lifetime of metastable state, Binder cumulant and hysteresis effect.

The damage spreading technique is useful tool to study the sensitivity of the system dynamics on the initial condition. A damage spreading analysis of semi-synchronous update leads to the conclusion that, the active phase is chaotic and the transition in this system is as same the frozen chaotic transition. The damage spreading inside the active phase (far from critical point) is compact. However, near the critical point the damage spreading is fractal.

5. Acknowledgments

MAS thanks Govt. of Yemen for scholarship. PMG thanks DST for financial assistance.

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