Dynamical Phases and Related Phase Transitions in Coupled Map Lattices

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Abstract

Complex systems are mainly collections of interacting subunits which result in the intricate global behaviours and patterns which can be sometimes inferred from studying individual elements, nature of interaction among them and their collective stability landscapes. The mathematical tools for studying pattern formation in nature can be classified in broad classes partial differential equation, coupled oscillators, cellular automata and coupled map lattices CML. In this work we study different coupled map lattice models.

Universality and scaling in critical many body systems have been studied since long time. The phase transition in Ising and Percolation models have been studied in depth. However, there are not many studies where the phenomenology of coupled map lattices has been used to see different phases and also characterize the transition as a nonequilibrium phase transition problem. The earlier attempts in CMLs have been mainly to identify a dynamical system showing transition of known universality classes like Ising or directed percolation. In this work we have studied coupled map lattices with various modifications like changing the local dynamics, varying the network topology, addition of delayed feedback and stochastically switching of the coupling type. In first chapter, we have presented a detailed literature survey of various themes discussed in the thesis. We are mainly interested in phase transition to a partially or fully absorbing state. Two of our studies are to fully absorbing state and one study related to partially arrested state.

We study the synchronization of network of coupled chaotic maps modelling neuronal activity, which is an absorbing state transition. We provide a brief literature survey of different networks, neuronal modeling and studies on coupled neurons. In our studies, the range of stability where the spatiotemporal fixed point gains stability is unchanged in presence of randomness in the connections. As coupling gets weaker, the spatiotemporal fixed point loses stability, and one obtains chaos. In this regime where the coupling connections are completely regular, the network becomes spatiotemporally chaotic. Interestingly however, in the presence of random links one obtains spatial spatial synchronization in the network. We find that this range if synchronized chaos increases exponentially with the fraction of random links in the network. Further, in the space of fixed coupling strengths, the synchronization transition occurs at a finite probability of rewiring, a scenario distinct from many other examples of synchronization transitions which occur at very small values of rewiring probability. Further, we show that the synchronization here is robust in the presence of parametric noise, namely in network of nonidentical neuronal maps. Finally we check the generality of our observations in networks displaying both spiking and bursting dynamics. We also provide analytical calculation to affirm the numerical results.

We also study a lattice model where the coupling stochastically switches between repulsive (subtractive) and attractive (additive) at each site with a probability p at every time instant. We observe that such a kind of coupling stabilizes the local fixed of a circle map, with resultant globally stable attractor providing a unique absorbing state. Interstingly, a continuous phase transition is observed from the absorbing state to spatiotemporal chaos via spatiotemporal

intermittency for range of values of p. It is interesting to note that the transition falls in class of directed percolation. Static and spreading exponents along with relevant scaling laws are found to be obeyed confirming the directed percolation universality class in spatiotemporal regime. The model was motivated from the field theoretic process leading to directed percolation behaviour. The stochastically switched coupling provides an dynamic analogue to percolation probability and the coupling type, i.e repulsive and attractive can be considered as analogous to particle creation and annihilation processes.

Delayed feedback is introduced in coupled circle map lattice with a motivation to increase the dimensionality of circle map. Various dynamical phases are observed in this system of coupled high-dimensional maps. We observe an interesting transition from localized chaos to spatiotemporal chaos. We study this transition as a dynamic phase transition. We observe that persistence acts as an excellent quantifier to describe this transition. Taking the location of the fixed point of circle map (which does not change with feedback) as a reference point, we compute number of sites which have been greater that (less than) the fixed point till time t. Though local dynamics is high dimensional in this case, this definition of persistence which tracks a single variable is an excellent quantifier for this transition. In most cases, we also obtain a well defined persistence exponent at the critical point and observe conventional scaling as seen in second order phase transitions. This indicates that persistence could work as good order parameter for transitions from fully or partially arrested phase. We also give an explanation based on perturbation theory for gaps in eigenvalue spectrum of Jacobian of localized state.

0.1 list of publications

1. Dynamic Phase Transition from localized to spatiotemporal chaos in coupled circle maps,
   A. R. Sonawane and P. M. Gade,
   Proc. 51 DAE Solid state Physics Symposium, 51, 129, 2006

2. Synchronization in a network of model Neurons,
   M. P. K. Jampa, **A. R. Sonawane**, P. M. Gade, S. Sinha,
   Phys. Rev. E, 75, 026215, 2007.

3. Directed percolation criticality due to stochastic switching between attractive and repulsive coupling in coupled circle maps,
   A. R. Sonawane,
   Phys. Rev. E, 81, 056206, 2010.

4. Dynamic Phase Transition from localized to spatiotemporal chaos in coupled circle maps with feedback.
   A. R. Sonawane and P. M. Gade,
   Chaos, accepted

0.2 Conferences and Workshops

1. Complex Systems School jointly organized by Santa-Fe Institute and Institute of Mathematical Sciences, Chennai, held at Chennai in Jan-2006, duration - 3 weeks.

2. ICTP Hands-on Research school on Complex Systems held by ICTP and IPR Gandhinagar at IPR in Jan 2008, duration - 2 weeks.

3. Dynamics on Networks held st IISER Pune. DEc 2009

4. 51 DAE Solid State Physics Symposium held at Bhopal, Dec-2006

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